

Gravitational waves from WZW models

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Abstract.

A brief review is given of the recent solution of a non-compact CFT describing a NS-supported pp-wave background. We will first explain how to compute the three and four-point correlators using current algebra techniques, thereby showing that some generic features of the non-compact WZW models become very clear in this simple context. We will then present the Penrose limit as a contraction of an $U(1) \times SU(2)_k$ WZW model, an approach that could prove useful in order to understand holography for pp-wave space-times. We will finally comment on the string amplitudes and on the existence of two flat space limits.

Plane-fronted gravitational wave backgrounds are exact solutions of the classical string equations of motion [1], a property that follows from the existence of a covariantly constant null vector in these space-times [2]. They represent a large class of time-dependent and possibly singular solutions of string theory that enable the investigation of some non-trivial aspects of the string dynamics by quantizing the world-sheet σ -model in the light-cone gauge (for a recent discussion see [3]). Whenever the σ -model displays a larger symmetry, such as a current algebra symmetry, we can study the properties of the corresponding pp-wave backgrounds in more detail and in particular we can expect to be able to compute some correlation functions. The first example of a gravitational wave related to a WZW model was discovered by Nappi and Witten [4]. They considered a WZW model based on the Heisenberg group H_4 , defined by the following commutation relations:

$$[P^+, P^-] = -2i\mu K, \quad [J, P^\pm] = \mp i\mu P^\pm. \quad (1)$$

Since the group is not semi-simple, the Killing form is degenerate but the stress-energy tensor can still be represented as a bilinear form in the currents,

$$T = \frac{1}{4}(P^+ P^- + P^- P^+) + JK + \frac{\mu^2}{2} K^2. \quad (2)$$

The background metric and the antisymmetric tensor field of the corresponding σ -model are:

$$ds^2 = -2dudv - \frac{\mu^2 r^2}{4} du^2 + dr^2 + r^2 d\varphi^2, \quad B_{\varphi u} = \frac{\mu r^2}{2}, \quad (3)$$

and describe a four-dimensional gravitational wave. Gravitational waves in $2 + 2n$ dimensions ($n \geq 1$) with similar metrics and fluxes correspond to WZW models based on the H_{2+2n} groups. Their central charge is $c = 2 + 2n$.

There are several reasons for being interested in this class of space-times. First of all they are one of the few examples of curved backgrounds where it has been possible to compute tree-level string amplitudes [5]. Moreover they display in a very clean context, due to their simple algebraic structure, all the new interesting features of the non-compact WZW models, such as an infinite number of representations and the spectral flow. Finally they arise as Penrose limits of space-times for which we know the holographic description and therefore we expect that the analysis of the string correlators could teach us something about the holographic description of the pp-waves [6]. The most studied case is the BMN limit of $\mathcal{N} = 4$ SYM [7] related to string theory in the maximally supersymmetric wave in ten dimensions [8]. In this example the gravitational wave is supported by a RR flux and the world-sheet σ -model has been quantized only in the light-cone gauge [9]. On the other hand, the H_4 WZW model can be quantized in a covariant way and the three and four-point amplitudes have been computed using standard current algebra techniques, as we will discuss in the following. This WZW model arises as the Penrose limit of the near-horizon region of a collection of NS5 branes and is therefore related to the Little String Theories [10]. An interesting application of our results would be the study of holography in the pp-wave limit of $AdS_3 \times S^3$ using the H_6 WZW model.

We start our analysis by discussing the spectrum of the H_4 WZW model. For this purpose we label the states with two quantum numbers: p , the eigenvalue of K which we can identify with the momentum conjugate to the v direction and j , the eigenvalue of J . If we make a Fourier transformation in the light-cone directions, the wave equation in the pp-wave background (3) reduces to the Schrödinger equation for a two-dimensional harmonic oscillator with frequency proportional to μp . As a consequence particles with non-zero p are confined by the gravitational wave in periodic orbits in the transverse plane while particles with $p = 0$ do not feel the potential and can move freely in the transverse plane. These three different types of motion parallel the three types of unitary representations of the H_4 algebra:

- (i) the lowest-weight $V_{p,\hat{j}}^+$ representations with positive light-cone momentum p and $j = \hat{j} + n$, $n \in \mathbb{N}$;
- (ii) the highest-weight $V_{p,\hat{j}}^-$ representations with negative light-cone momentum $-p$ and $j = \hat{j} - n$, $n \in \mathbb{N}$;
- (iii) the $V_{s,\hat{j}}^0$ representations with zero light-cone momentum and $j = \hat{j} + n$, $n \in \mathbb{Z}$. The positive number s measures the radial momentum in the transverse plane.

The highest-weight representations of the current algebra lead to a ghost-free physical string state spectrum only if we restrict the unitary representations of the zero-mode algebra to the range $0 \leq \mu p < 1$. This bound has the same origin as the bound on the quantum number l labeling the highest-weight representations of $SL(2, \mathbb{R})_k$ and

it is therefore not surprising that also in our case the states with $\mu p \geq 1$ belong to the so-called spectral-flowed representations [11]. These representations are highest-weight representations of the isomorphic algebra $\tilde{H}_{4,w}$, $w \in \mathbb{Z}$, whose modes are related to the original ones by

$$\tilde{P}_n^\pm = P_{n \mp w}^\pm, \quad \tilde{K}_n = K_n - iw\delta_{n,0}, \quad \tilde{J}_n = J_n, \quad \tilde{L}_n = L_n - iwJ_n. \quad (4)$$

The complete spectrum of the model then contains the $V_{p,\hat{j}}^+$ representations, with $\mu p < 1$ and $\hat{j} \in \mathbb{R}$, and their spectral-flowed images $\Omega_w(V_{p,\hat{j}}^+)$ with $w \in \mathbb{N}$; the $V_{p,\hat{j}}^-$ representations, with $\mu p < 1$ and $\hat{j} \in \mathbb{R}$, and their spectral-flowed images $\Omega_{-w}(V_{p,\hat{j}}^-)$ with $w \in \mathbb{N}$; the $V_{s,\hat{j}}^0$ representations, with $s \geq 0$ and $\hat{j} \in [-1/2, 1/2)$, and their spectral-flowed images $\Omega_w(V_{s,\hat{j}}^0)$ with $w \in \mathbb{Z}$. We consider left-right symmetric combination of the representations with the same amount of spectral flow in the two sectors. Note that in string theory all the states with $\mu p \in \mathbb{Z}$ can move freely in the transverse plane, not only the states with $p = 0$. These states are the so-called long strings [12, 11].

Let us now concentrate on the highest-weight representations. We introduce the local primary fields $\Phi_q^a(z, \bar{z}; x, \bar{x})$ that depend on the world-sheet coordinates z, \bar{z} and on two further variables x and \bar{x} , that encode the states of the left and right infinite-dimensional representations of the left and right H_4 current algebras. In the following expressions we will often omit the dependence on \bar{z} and \bar{x} . Here a labels the different representations ($a \in \{+, -, 0\}$) and q stands for the set of charges needed to completely specify a given representation, that is $q = (p, \hat{j})$ when $a = \pm$ and $q = (s, \hat{j})$ when $a = 0$.

The introduction of the auxiliary charge variable x is a very useful tool [13]. The zero-modes of the currents can be realized as operators acting on these variables and the OPE's can be written in a clear and compact form. In our case, we use the standard realization of the H_4 group in terms of multiplication and differentiation operators. For the $V_{p,\hat{j}}^\pm$ representations we introduce the fields

$$\Phi_{p,\hat{j}}^\pm(z, x) = \sum_{n=0}^{\infty} R_{p,\hat{j};n}^\pm(z) \frac{(x\sqrt{p})^n}{\sqrt{n!}}, \quad (5)$$

and the operators

$$P_0^\pm = \sqrt{2}p \, x, \quad P_0^\mp = \sqrt{2} \, \partial_x, \quad J_0 = i(\hat{j} \pm x\partial_x), \quad K_0 = \pm ip. \quad (6)$$

The monomials $b_n = \frac{(x\sqrt{p})^n}{\sqrt{n!}}$ form an orthonormal basis if we define the scalar product using a gaussian measure. The right-moving algebra is similarly realized as an algebra of operators acting on the independent variable \bar{x} . The conformal dimension of the primary fields is

$$h = \mp p\hat{j} + \frac{\mu p}{2}(1 - \mu p). \quad (7)$$

For the $V_{s,\hat{j}}^0$ representations we introduce the fields

$$\Phi_{s,\hat{j}}^0 = \sum_{n=-\infty}^{\infty} R_{s,\hat{j};n}^0(z) x^n, \quad (8)$$

with $x = e^{i\alpha}$ and the operators

$$P_0^+ = sx, \quad P_0^- = \frac{s}{x}, \quad J_0 = i(\hat{j} + x\partial_x), \quad K_0 = 0. \quad (9)$$

The conformal dimension of the primary fields is $h = s^2/2$. The general form of the three-point couplings can now be written as

$$\langle \Phi_{q_1}^a(z_1, x_1) \Phi_{q_2}^b(z_2, x_2) \Phi_{q_3}^c(z_3, x_3) \rangle = \frac{\mathcal{C}_{abc}(q_1, q_2, q_3) D_{abc}(x_1, x_2, x_3)}{\prod_{j>i=1}^3 |z_{ij}|^{2(2h_i+2h_j-h)}}, \quad (10)$$

where $h = \sum_{i=1}^3 h_i$. Here the $\mathcal{C}_{abc}(q_1, q_2, q_3)$ are the quantum structure constants of the CFT and the functions D_{abc} are the generating functions for the Clebsch-Gordan coefficients of the left and right H_4 algebras. As an explicit example consider the fusion between two Φ^+ representations

$$[\Phi_{p_1, \hat{j}_1}^+] \otimes [\Phi_{p_2, \hat{j}_2}^+] = \sum_{n=0}^{\infty} [\Phi_{p_1+p_2, \hat{j}_1+\hat{j}_2+n}^+]. \quad (11)$$

The function D_{++-} is given by

$$D_{++-}(x_1, x_2, x_3) = \left| e^{-x_3(p_1x_1+p_2x_2)} (x_2 - x_1)^{-L} \right|^2, \quad (12)$$

and is non-zero only when $p_3 = p_1 + p_2$ and $L = \sum_{i=1}^3 \hat{j}_i$ is a non-positive integer.

The quantum structure constants $\mathcal{C}_{abc}(q_1, q_2, q_3)$ can be derived studying the factorization of the four-point amplitudes [5], as we will illustrate momentarily. There are three types of couplings:

1) Couplings of the form $\langle V^\pm V^\pm V^\mp \rangle$, involving only V^\pm representations. For example

$$\mathcal{C}_{++-}(q_1, q_2, q_3) = \frac{1}{\Gamma(1 - \hat{j}_1 - \hat{j}_2 - \hat{j}_3)} \left[\frac{\gamma(p_3)}{\gamma(p_1)\gamma(p_2)} \right]^{\frac{1}{2} - \hat{j}_1 - \hat{j}_2 - \hat{j}_3}, \quad (13)$$

where $\gamma(x) = \Gamma(x)/\Gamma(1-x)$. The other couplings can be obtained from (13) using the symmetry of the \mathcal{C}_{abc} in their indexes and the fact that $\mathcal{C}_{++-} = \mathcal{C}_{--+}$, up to inverting the sign of all the \hat{j}_i . For example

$$\mathcal{C}_{+--}(q_1, q_2, q_3) = \frac{1}{\Gamma(1 + \hat{j}_3 + \hat{j}_1 + \hat{j}_2)} \left[\frac{\gamma(p_1)}{\gamma(p_2)\gamma(p_3)} \right]^{\frac{1}{2} + \hat{j}_3 + \hat{j}_1 + \hat{j}_2}, \quad (14)$$

with $p_3 = p_1 + p_2$ and $\hat{j}_3 = -\hat{j}_1 - \hat{j}_2 + n$, $n \in \mathbb{N}$. We will often use a short-hand notation for these three-point couplings writing for instance $\mathcal{C}_{++-}(q_1, q_2, n)$ to denote the coupling in equation (13) with $\hat{j}_3 = -\hat{j}_1 - \hat{j}_2 - n$.

2) Couplings of the form $\langle V^+ V^- V^0 \rangle$. They are given by

$$\mathcal{C}_{+-0}(p, \hat{j}_1; p, \hat{j}_2; s, \hat{j}_3) = e^{\frac{s^2}{2}[\psi(p) + \psi(1-p) - 2\psi(1)]}, \quad (15)$$

where $\psi(x) = \frac{d \ln \Gamma(x)}{dx}$ is the digamma function. We introduce a short-hand notation also for these couplings setting $\mathcal{C}_{+-0}(p, s) \equiv \mathcal{C}_{+-0}(p, \hat{j}_1; p, \hat{j}_2; s, \hat{j}_3)$.

3) Couplings between three V^0 representations. They are the same as in flat space.

We now turn to the four-point amplitudes and explain how they can be computed using factorization and the Knizhnik-Zamolodchikov (KZ) equations. As it is well-known the correlation functions between the affine primary fields of a WZW model

satisfy a system of differential equations, the KZ equations, as a direct consequence of the Sugawara form of the stress-energy tensor. For a four-point amplitude the global conformal and H_4 symmetries can be used to reduce the dependence on the eight z_i and x_i variables to the dependence on only two variables, the cross-ratio $z = \frac{z_{12}z_{34}}{z_{13}z_{24}}$ and an invariant x that is different for different types of correlators. In this way the KZ equation becomes a partial differential equation in two variables. Using the operator algebra the four-point amplitudes can be decomposed in a sum over intermediate representations of the affine algebra. The functions that appear in this decomposition are called conformal blocks and are particular solutions of the KZ equation which satisfy suitable boundary conditions. The four-point amplitudes can then be reconstructed as a monodromy invariant combination of the conformal blocks. The requirement of monodromy invariance arises because the four-point functions can be factorized in different ways and all of them must agree in order to respect the associativity of the operator algebra. In terms of the cross-ratio z , the three factorization limits are $z = 0$, 1 and ∞ .

Here we discuss only the general structure of the amplitudes. Explicit expressions can be found in [5]. The simplest four-point functions involve three Φ^+ and one Φ^- vertex operators

$$\langle \Phi_{p_1, \hat{j}_1}^+(z_1, x_1) \Phi_{p_2, \hat{j}_2}^+(z_2, x_2) \Phi_{p_3, \hat{j}_3}^+(z_3, x_3) \Phi_{p_4, \hat{j}_4}^-(z_4, x_4) \rangle, \quad (16)$$

with $p_1 + p_2 + p_3 = p_4$ and $L = \sum_{i=1}^4 \hat{j}_i \in \mathbb{Z}$. These amplitudes factorize on a finite number of conformal blocks when $L \leq 0$ and vanish when $L > 0$. In the s -channel the intermediate states belong to the representations $\Phi_{p_1+p_2, \hat{j}_1+\hat{j}_2+n}^+$ with $n = 0, 1, \dots, |L|$ and the four-point functions can be written as

$$\mathcal{A}(z, \bar{z}, x, \bar{x}) = \sum_{n=0}^{|L|} \mathcal{C}_{++-}(q_1, q_2, n) \mathcal{C}_{+-+}(q_3, q_4, |L| - n) |\mathcal{F}_n(z, x)|^2. \quad (17)$$

The most interesting correlators involve two Φ^+ and two Φ^- vertex operators

$$\langle \Phi_{p_1, \hat{j}_1}^+(z_1, x_1) \Phi_{p_2, \hat{j}_2}^-(z_2, x_2) \Phi_{p_3, \hat{j}_3}^+(z_3, x_3) \Phi_{p_4, \hat{j}_4}^-(z_4, x_4) \rangle, \quad (18)$$

with $p_1 + p_3 = p_2 + p_4$. In this case the number of intermediate states is infinite and the amplitudes can be written as

$$\mathcal{A}(z, \bar{z}, x, \bar{x}) = \sum_{n=0}^{\infty} \mathcal{C}_{+-}(q_1, q_2, n) \mathcal{C}_{-+}(q_3, q_4, n + |L|) |\mathcal{F}_n(z, x)|^2. \quad (19)$$

Here the conformal blocks correspond to the representations $\Phi_{p_1-p_2, \hat{j}_1+\hat{j}_2-n}^+$ and we have assumed $p_1 > p_2$ and $L \leq 0$. When $p_1 = p_2$ the intermediate states belong to the $\Phi_{s, \hat{j}_1+\hat{j}_2}^0$ representations and the amplitude factorizes as follows

$$\mathcal{A}(z, \bar{z}, x, \bar{x}) = \int_0^\infty ds s \mathcal{C}_{+-0}(q_1, q_2, s) \mathcal{C}_{-+0}(q_3, q_4, s) |\mathcal{F}_s(z, x)|^2. \quad (20)$$

Finally when $p_1 + p_3 \geq 1$ we can explicitly verify by studying the factorization of the amplitude that the intermediate states belong to the spectral-flowed representations. The inclusion of these representations is therefore necessary in order to define a closed

operator algebra. The fusion rules between the spectral-flowed representations turn out to be

$$\Omega_{w_1}(\Phi_1) \otimes \Omega_{w_2}(\Phi_2) = \Omega_{w_1+w_2}(\Phi_1 \otimes \Phi_2) , \quad (21)$$

as first proposed in [14]. A detailed analysis of the factorization properties of the four-point amplitudes and of the transformations of the conformal blocks can be found in [5]. It would be interesting to compare the braiding and fusion matrices that exchange the different basis of conformal blocks with the 6j-symbols of the quantum Heisenberg group.

There is another interesting aspect of the model we would like to briefly mention. The H_4 current algebra has a free field realization [15] in term of which the primary vertex operators correspond to orbifold twist fields. As a consequence the H_4 three-point couplings displayed before can be compared with the couplings computed in [16] for the case of a rational twist. Similarly the H_4 four-point correlators just discussed can be considered as generating functions for correlators between arbitrarily excited twist fields. Finally the three and four-point correlators of the H_4 WZW model (and in principle also arbitrary N -point correlators) can be evaluated using a Wakimoto representation, as recently discussed in [17].

The Nappi-Witten gravitational wave can be obtained as the Penrose limit of a space of the form $\mathbb{R} \times S^3$. From the world-sheet point of view this operation amounts to a contraction of the affine algebra $U(1)_{\text{time}} \times SU(2)_k$ [18]. In the contraction, the level k is sent to infinity and the quantum numbers of the states are scaled in such a way that the representations of the original algebra organize themselves in representations of the contracted one. Since it is by now well known how to perform the Penrose limit in space-time, we will concentrate on the contraction of the current algebra. Let J^0 be the $U(1)$ current and J^3, J^\pm the three $SU(2)$ currents. The contraction to the H_4 algebra is obtained by first changing basis to the new currents

$$K(z) = \frac{2i}{k} J^0(z) , \quad J(z) = i(J^0(z) - J^3(z)) , \quad P^\pm(z) = \sqrt{\frac{2}{k}} J^\pm(z) , \quad (22)$$

and then sending the level k to infinity.

As we said, the first thing we would like to understand is how the states of the original and of the contracted models are related. There are three different classes of states that are relevant in the limit. Let us label the original states with their spin l and with their J^0 and J^3 eigenvalues, q and m respectively. The quantum numbers of the first two classes of states scale as

$$q = \pm \frac{kp}{2} , \quad l = \frac{kp}{2} \mp \hat{j} , \quad m = \pm \frac{kp}{2} - \hat{j} \mp n . \quad (23)$$

These states in the limit are very close either to the top or to the bottom of an $SU(2)$ representation and give rise to the $V_{p,\hat{j}}^\pm$ representations. The quantum numbers of the third class of states scale as $l \sim \sqrt{k}s, q, m \sim O(1)$ and correspond to states in the middle of an $SU(2)$ representation. In the limit they give rise to the $V_{s,\hat{j}}^0$ representations. At

the level of the semiclassical wave functions the contraction amounts to a limit relating the Jacobi and Laguerre polynomials.

Having understood how the states are connected, we can study how dynamical quantities such as the three and the four-point functions behave in the limit. The $SU(2)_k$ WZW model was solved by Fateev and Zamolodchikov in [13]. They introduced a charge variable x for $SU(2)$ and studied correlation functions between affine primary fields $\Phi_l(z, x)$. The $SU(2)$ Ward identities completely fix the dependence of the three-point couplings on the x variables: $D(l_1, l_2, l_3) = \prod_{i < j}^3 x_{ij}^{l_{ij}}$ with $l_{12} = l_1 + l_2 - l_3$ and cyclic permutations. The quantum structure constants of the operator algebra are

$$C^2(l_1, l_2, l_3) = \gamma \left(\frac{1}{N} \right) P^2(l_1 + l_2 + l_3 + 1) \prod_{i=1}^3 \frac{P^2(l_1 + l_2 + l_3 - 2l_i)}{\gamma \left(\frac{2l_i + 1}{N} \right) P^2(2l_i)}, \quad (24)$$

with

$$P(n) = \prod_{m=1}^n \gamma \left(\frac{m}{N} \right), \quad P(0) = 1, \quad N = k + 2. \quad (25)$$

It is easy to verify that if we use the following relations between the H_4 and $U(1) \times SU(2)_k$ primary fields

$$\Phi_{p,\hat{j}}^-(x, z) = \lim_{k \rightarrow \infty} \Phi_l \left(\sqrt{\frac{p}{2l}} x, z \right), \quad 2l = kp + 2\hat{j}, \quad (26)$$

$$\Phi_{p,\hat{j}}^+(x, z) = \lim_{k \rightarrow \infty} \left(\frac{1}{x} \sqrt{\frac{2l}{p}} \right)^{-2l} \Phi_l \left(\frac{1}{x} \sqrt{\frac{2l}{p}}, z \right), \quad 2l = kp - 2\hat{j}, \quad (27)$$

$$\Phi_s^0(x, z) = \lim_{k \rightarrow \infty} x^{-l-\hat{j}} \Phi_l(x, z), \quad 2l = \sqrt{2k} s, \quad (28)$$

the $U(1) \times SU(2)_k$ three-point couplings in equation (24) reproduce the H_4 couplings displayed in equations (13), (14) and (15). A discussion of the behaviour in the limit of the simplest four-point functions and of the spectral flow can be found in [5].

We are ultimately interested in critical string theory backgrounds of the form $C = C_{H_4} \times C_{\text{int}} \times C_{\text{gh}}$, the simplest choice being $C_{\text{int}} = \mathbb{R}^{22}$. The string theory amplitudes are given by the CFT correlators integrated over the moduli space of the corresponding surface with punctures. For the four-point functions this means that we have to integrate over the cross-ratio z . The most interesting feature of the string amplitudes in this pp-wave background is that the propagation of long string states in the intermediate channels gives rise to a logarithmic branch cut.

The last aspect we would like to discuss is the flat space limit of the pp-wave metric [5]. The value of the parameter μ in equation (3) can be changed performing a boost $u \rightarrow \lambda u$, $v \rightarrow v/\lambda$. There are however two interesting limits to consider: $\mu \rightarrow 0$ and $\mu \rightarrow \infty$. In the limit $\mu \rightarrow 0$ the metric reduces to the metric of flat Minkowski space and the current algebra to the algebra of four free bosons. We want to emphasize that this limit can be considered as a contraction of the algebra H_4 to $U(1)^4$ and can be discussed in exactly the same way as we did for the contraction from $U(1) \times SU(2)$ to

H_4 . Note that these contractions are singular limits and in particular they change the asymptotic structure of the space-time. At the level of the semiclassical wave-functions the contraction $\mu \rightarrow 0$ amounts to a limit relating the Laguerre polynomial and the Bessel functions.

In the $\mu \rightarrow 0$ limit we obtain flat space as suggested by the classical intuition: the potential flattens and the states trapped by the wave describe larger and larger orbits until they become free. All the states in flat space arise from states in the highest-weight representations of the H_4 WZW model, while the spectral-flowed representations are scaled out of the spectrum. We would like to point out that also the limit $\mu \rightarrow \infty$ leads to flat space even though the states that survive in the limit are markedly different. Indeed in this case the flat space vertex operators arise from the spectral-flowed continuous representations, while all the other representations with $\mu p \notin \mathbb{Z}$ are so strongly trapped by the potential that disappear from the spectrum. This is a typical stringy mechanism and it is similar under some respect to the large and small radius limit of a compactified boson.

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